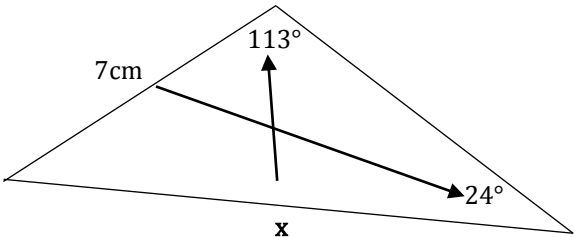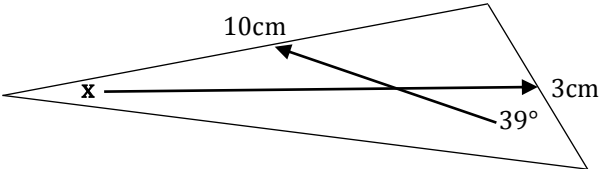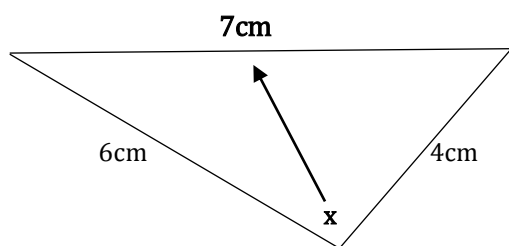# Basic Skills

Unit 2 – Expressions and Formulae

Questions	Answers
1) Finding the area of a triangle. Calculate the area of the triangle below. Give your answer to two significant figures. 4cm 31° 5cm	Area = ½ a b sin C $\quad$ = ½ × 4 × 5 × sin 31 $\quad$ = 5.1503........ $\quad$ = 5.2cm² to 2 s.f. This formula is given to you on the front of the exam paper. Notice that you need to know the length of two sides and the angle in between to use this formula.
2) Calculating the length of a side using cosine rule,. Calculate the length of the side marked **x** in the triangle below. Give your answer to three significant figures. 6cm 38° x 4cm	$x^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \times \cos 38$ $\quad$ = 14.175....... $x = \sqrt{14.175....}$ $\quad$ = 3.765.... = 3.77cm to 3 s.f. Cosine rule is given in the exam ($a^2 = b^2 + c^2 - 2bc \cos A$). Notice that, in order to use the cosine rule, you need to know the length of two sides and the angle between (like the area formula above).
3) Calculating the length of a side using the sine rule. Calculate the length of the side marked **x** in the triangle below. Give your answer to three significant figures. 113° 7cm 24° x	$\dfrac{x}{\sin(113)} = \dfrac{7}{\sin(24)}$ $x = \dfrac{7 \sin(113)}{\sin(24)} = 15.842..... = 15.8$cm to 3 s.f. The sine rule is given in the exam ($\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$). Notice that, in order to use the sine rule, you need to know the angle opposite the side you are trying to find and another angle/side pair.
4) Calculating the size of an angle using the sine rule Calculate the size of the angle marked **x** in the triangle below. Give your answer to three significant figures. 10cm x $\quad$ 3cm 39°	$\dfrac{\sin x}{3} = \dfrac{\sin(39)}{10}$ $\sin x = \dfrac{3 \sin(39)}{10} = 0.1887....$ $x = 10.8825.... = 10.9°$ to 3 s.f. Calculating the angle is slightly easier if you turn the sine rule upside down. It is given as ($\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$) in the exam. Notice that, in order to use the sine rule, you need to know the side opposite the angle you are trying to find and another angle/side pair.

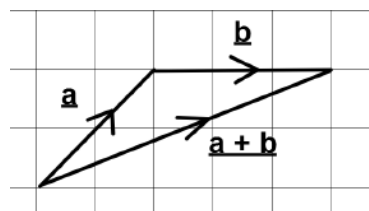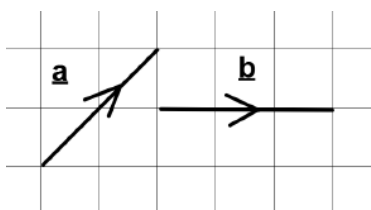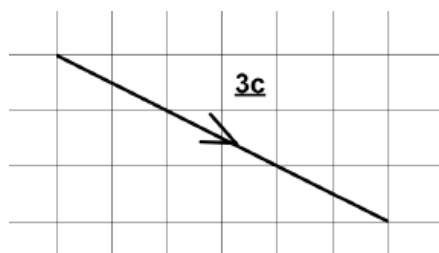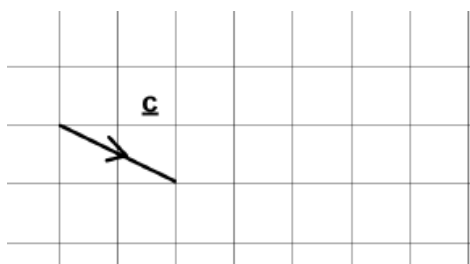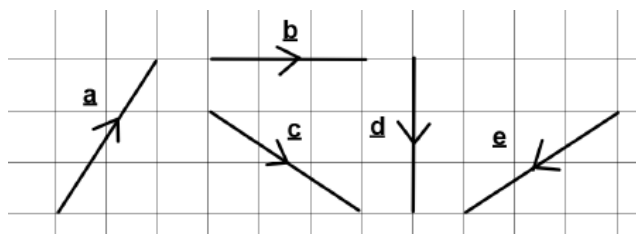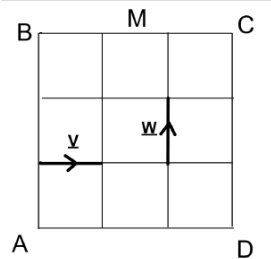|	
5) Calculate the size of an angle using the cosine rule. Calculate the size of the angle marked **x** in the triangle below. Give your answer to three significant figures. 7cm 6cm 4cm x	$\cos x = \dfrac{6^2 + 4^2 - 7^2}{2 \times 6 \times 4} = 0.0625$ $x = 86.4166\ldots = 86.4°$ to 3 s. f. The version of the cosine rule for finding an angle is given in the exam ($\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$). Notice that, in order to use the cosine rule to find an angle, you need to know the length of all three sides. The side opposite the angle you wish to find appears only once at the end of the top line.
6) Adding vectors Vectors **a** and **b** are shown below. Draw **a** + **b**.	
7) Multiplying vectors by a number. Vector **c** is shown below. Draw **3c**.	
8) Writing components of vectors Write down the components of these vectors.	$\underline{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ $\underline{c} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $\underline{d} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$ $\underline{e} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$
9) Adding vectors in component form. If $\underline{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$, calculate **a** + **b**.	$\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$
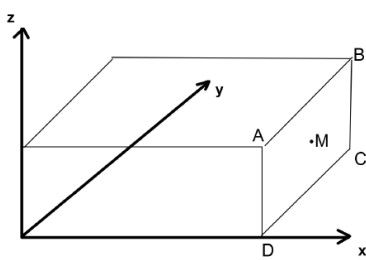
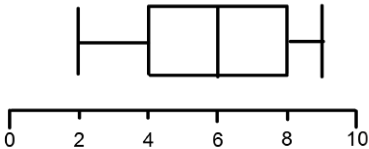|	
10) Subtracting vectors in component form. If $\underline{c} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\underline{d} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$, calculate $3\underline{c} - 2\underline{d}$.	$3\begin{pmatrix} 2 \\ 3 \end{pmatrix} - 2\begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \end{pmatrix} - \begin{pmatrix} -4 \\ -2 \end{pmatrix} = \begin{pmatrix} 10 \\ 11 \end{pmatrix}$
11) Finding the magnitude of a vector. If $\underline{e} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$ and $\underline{f} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$, calculate $\lvert 2\underline{e} + 3\underline{f} \rvert$	$2\underline{e} + 3\underline{f} = 2\begin{pmatrix} -3 \\ 0 \end{pmatrix} + 3\begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \end{pmatrix} + \begin{pmatrix} 12 \\ 12 \end{pmatrix} = \begin{pmatrix} 6 \\ 12 \end{pmatrix}$ $\lvert 2\underline{e} + 3\underline{f} \rvert = \sqrt{6^2 + 12^2} = \sqrt{180} = 6\sqrt{5}$ (or 13.4164….) The magnitude is the length of the vector. You may just give the answer as a decimal or you may be asked to simplify the surd.
12) Using vectors to define a pathway. If M is the midpoint of BC, express $\overrightarrow{DM}$ in terms of vectors $\underline{v}$ and $\underline{w}$. 	To get from D to M using only the vectors given:- $$\overrightarrow{DM} = -1\tfrac{1}{2}\,\underline{v} + 3\underline{w}$$ This could also be written as:- $$\overrightarrow{DM} = 3\underline{w} - \frac{3}{2}\,\underline{v}$$ Remember that you can use a negative to reverse the direction of a vector or you can use multiples of the given vectors.
13) Vectors in three dimensions If $\underline{n} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and $\underline{m} = \begin{pmatrix} -1 \\ 0 \\ -2 \end{pmatrix}$, calculate $\lvert 3\underline{n} + 4\underline{m} \rvert$.	$3\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + 4\begin{pmatrix} -1 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix} + \begin{pmatrix} -4 \\ 0 \\ -8 \end{pmatrix} = \begin{pmatrix} 2 \\ 9 \\ -5 \end{pmatrix}$ $\lvert 3\underline{n} + 4\underline{m} \rvert = \sqrt{2^2 + 9^2 + (-5)^2} = \sqrt{110} = 10.488…..$ Notice that addition, multiplication and finding the magnitude of a 3D vector work in exactly the same way as a 2D vector, only there is one more number in each set of brackets.
14) Coordinates in three dimensions. The cuboid below has one vertex at (0,0,0). It is 10 units long, 6 units wide and 4 units high. M is the centre of the face ABCD. Write down the coordinates of M. 	M = (10, 3, 2) 10 along the x axis to reach the end of the cuboid at D 3 along DC to reach a point below M (y direction) 2 up to get to M (z direction)
15) Calculating percentage increase (or decrease) In a year the population of a town increased from 4567 to 5004. Work out the percentage increase. Give your answer to two significant figures.	Increase $= 5004 - 4567 = 428$ % Increase $= \frac{428}{4567} = 0.09371…. = 9.4\%$ to 2 s.f. Always use original amount

16) Calculating repeated percentage increase or decrease. **a)** Jim leaves £5000 in a bank account for 3 years at 2.4% interest. How much money will be in his account at the end of the 3 year period? Give your answer to the nearest penny. **b)** The number of pupils in a school is 780 but it likely to fall by 8% a year. If this is true how many pupils will there be in the school in five years? Round your answer to the nearest ten.	**a)** Amount in account $= 5000 \times 1.024^3 = 5368.70912$ $\qquad\qquad\qquad = £5368.71$ $\qquad\qquad\qquad$ to the nearest penny (1.024 because $1 \times$ the previous year plus 0.024 (2.4%)) **b)** Number of pupils $= 780 \times 0.92^5 = 514.0835\dots\dots$ $\qquad\qquad\qquad = 510$ pupils $\qquad\qquad\qquad$ to the nearest ten (0.92 because if 8% leave 92% remain)
17) Reversing a percentage increase or decrease. After a rise of 4% Jane earns £58 240 a year. What did she earn before her pay was increased?	Pay before $= 58\ 240 \div 1.04 = £56\ 000$ Consider how her pay was increased (by multiplying by 1.04) and reverse this change (by dividing by 1.04).
18a) Adding fractions $\qquad\qquad \frac{3}{8} + \frac{1}{4}$	Remember to use "kiss-and-smile". $\frac{3}{8} \times \frac{1}{4} = \frac{3 \times 4 + 1 \times 8}{8 \times 4} = \frac{12+8}{30} = \frac{20}{32} = \frac{5}{8}$ Always check – can your answer be simplified?
18b) Subtracting fractions $\qquad\qquad \frac{3}{5} - \frac{1}{10}$	Remember to use "kiss-and-smile". $\frac{3}{5} \times \frac{1}{10} = \frac{3 \times 10 - 5 \times 1}{5 \times 10} = \frac{30-5}{50} = \frac{25}{50} = \frac{1}{2}$ Always check – can your answer be simplified?
18c) Multiplying fractions $\qquad\qquad \frac{5}{9} \times \frac{3}{10}$	Multiply tops together, bottoms together. $\frac{5}{9} \times \frac{3}{10} = \frac{5 \times 3}{9 \times 10} = \frac{15}{90} = \frac{3}{18} = \frac{1}{6}$ Remember to simplify your answer or simplify before multiplying. $\frac{5^1}{9_3} \times \frac{3^1}{10_2} = \frac{1}{6}$
18d) Dividing fractions $\qquad\qquad \frac{3}{5} \div \frac{3}{4}$	Turn the second fraction upside down and multiply. $\frac{3}{5} \div \frac{3}{4} = \frac{3}{5} \times \frac{4}{3} = \frac{12}{15} = \frac{4}{5}$ Always check – can your answer be simplified?
19) Mixed numbers $\qquad\qquad 1\frac{3}{5} + 1\frac{1}{4}$	Convert to improper (top-heavy) fractions and proceed as above. $1\frac{3}{5} + 1\frac{1}{4} = \frac{8}{5} \times \frac{5}{4} = \frac{32+25}{20} = \frac{57}{20} = 2\frac{17}{20}$
20) Remember to apply order of operations with fractions $\qquad\qquad \frac{3}{4}\left[\frac{3}{7} + \frac{1}{3}\right]$	$\frac{3}{7} \times \frac{1}{3} = \frac{9+7}{21} = \frac{16}{21}$ $\frac{3}{4} \times \frac{16}{21} = \frac{48}{84} = \frac{4}{7}$ Remember BIDMAS – do the bit in brackets first.

21) Quartiles
 	
Write down the median and quartiles for this data.
 	
4 5 5 5 6 6 7 8 8 9 9 9 11 12
	4 5 5 5 6 6 7↑8 8 9 9↑ 9 11 12
	
	       ↑
	   Q_1 Median Q_3
	
	Median = 7.5, Q_1 = 5, Q_3 = 9

22) Semi-interquartile range If $Q_1 = 78$ and $Q_3 = 92$ what is the semi-interquartile range?	Semi -interquartile range (SIQR) $= \frac{Q_3 - Q_1}{2} = \frac{92-78}{2} = 7$
23) Boxplot Draw a boxplot for the following data:- 2 3 4 5 5 6 7 7 8 9 9	Lowest = 2, Q_1 = 4, Median = 6, Q_3 = 8, Highest =9
24) Mean Find the mean of the following numbers:- 45 49 55 66 78 88 92 94	Total $= 45 + 49 + 55 + 66 + 78 + 88 + 92 + 94 = 567$ Mean $= 567 \div 8 = 70.875$ (Divided by 8 because there were 8 numbers in the list) (Mean is sometimes labelled as $\bar{x}$ and $\bar{x} = \frac{\sum x}{n}$)
25) Standard deviation Calculate the standard deviation for the data below:- 21 24 25 30 35	Total $= 21 + 24 + 25 + 30 + 35 = 135$ Mean $= 135 \div 5 = 27$

Table for question 25:

x	**x - x̄**	**(x - x̄)²**
21	$21 - 27 = -6$	$(-6)^2 = 36$
24	$24 - 27 = -3$	$(-3)^2 = 9$
25	$25 - 27 = -2$	$(-2)^2 = 4$
30	$30 - 27 = 3$	$3^2 = 9$
35	$35 - 27 = 8$	$8^2 = 64$
		$\sum (x - \bar{x})^2 = 122$

Standard Deviation $= \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\frac{122}{4}} = 5.522.....$

(Remember the formula for standard deviation is given to you in the exam. In the formula n is the number of pieces of data given and $\bar{x}$ is the mean.)

26) Write appropriate comments on mean and standard deviation Group 1 – Mean height 172cm, Standard deviation 6.7 cm Group 2 – Mean height 181cm, Standard deviation 9.2 cm	**On average** Group 2 were taller because their mean height was greater than Group 1. Group 2 had a greater spread of heights because their standard deviation was greater than Group 1. (Mean comment should refer to the context i.e. **on average** they are faster/taller/lighter/more expensive/eat less etc.) (Standard deviation comment should mention greater or lesser spread of values.) (Both comments should also indicate clearly which group you are referring to and which statistics you used to draw your conclusion)